Formation, Assurance, and the Price of Commitment

Demand, screening, and credible resources in shared investment

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Abstract

Sufficient demand does not guarantee that an investment forms. Equal terms give a transparent bound on acceptable memberships, but conditional acceptance leaves refusal equilibria. With known willingness, the reserve infimum minimizes total subscription expense less the largest individual expense over forming coalitions. With private willingness, assurance must also screen: bonuses large enough to induce subscription can attract investors who prefer failure. A closed-cohort characterization gives exact bounds and reserve costs; continuous types at the acceptance margin yield an impossibility. A separate information-rent calculation identifies the interim commitment needed for efficient formation and establishes a uniform-cost pool-size threshold.

Keywords: investment formation; assurance contracts; private information; screening; participation; capital assembly.
JEL: D82, D86, G11, G24, G32.

John Komkov has commercial interests in Megalith and Argosy, which concern investment organization. AI tools assisted research, calculations, drafting, and internal review. Computational experiments in this note are synthetic; no investor data or new AI evaluation is used. Code, exact-arithmetic certificates, source comparisons, and the internal review record accompany this working paper.

1 From choosing investments to forming them

Portfolio choice asks which investments an investor should hold. Formation asks whether the investments he would like to hold can exist on terms that other investors will accept. A shared investment may have enough willing investors and still fail: each can prefer to wait for the others, a promised reward can attract the wrong participants, or the capital behind accepted commitments may be unavailable at settlement. A useful formation law must identify the demand that makes an investment possible, the contract that turns that demand into participation, and the resources that make the contract credible.

The distinction belongs within the economics of capital assembly and contracting, rather than outside existing finance. Halac, Kremer, and Winter (2020) study the payments needed to assemble heterogeneous investments with a unique outcome. Assurance and refund-bonus mechanisms already address refusal and strategic risk (Tabarrok, 1998; Zubrickas, 2014; Cason and Zubrickas, 2017). This note fixes successful investment terms and asks what failure-only payments can accomplish when willingness is either known or private. Fixing that contract class makes a precise accounting possible and exposes a screening constraint that a participation count conceals.

There are three linked results. First, with known strict willingness and an arbitrary upward-closed formation technology, the reserve infimum is

$$R_{\mathrm{sel}}^{\mathrm{inf}} = \min_{F \in \mathcal{F}} \left(\sum_{i \in F} h_i - \max_{i \in F} h_i \right). \tag{1}$$

Here h_i is the expense of subscribing, and $\mathcal{F}$ is the family of memberships that form the investment. Every member of a coalition minimizing this reserve expression except one can be induced to subscribe by a payment slightly above his expense if formation fails. The last member then completes the coalition. A converse establishes the least credible promise, including mixed equilibria and all ties.

Second, private willingness puts an upper bound on the same payment. In a cohort in which every investor is indispensable, let investor i gain $g_i > 0$ from success when willing and lose $\ell_i > 0$ when unwilling. Let A_i be the probability that all the other investors are willing, and b_i his failure payment net of expense. Exact screening in every Bayesian equilibrium requires

$$-\frac{A_i g_i}{1 - A_i} < b_i < \frac{A_i \ell_i}{1 - A_i}, \tag{2}$$

with at most one nonpositive b_i. The lower bound protects willing participation; the upper bound deters bonus-seeking by unwilling types. This gives a reserve formula for the private-type cohort. When unwilling types approach indifference continuously, the screening ceiling tends to zero and the two objectives cannot both be achieved by fixed failure bonuses. This is a sharp boundary for the declared contract class, not an impossibility for all mechanisms.

Third, a different notion of commitment arises when investors can enroll before learning their private costs and accept negative utility at some later types. The minimum aggregate relaxation of interim participation is information rents less surplus, when positive. This is an application of payment equivalence, directly benchmarked against Krishna and Perry (1998). For uniform costs and value equal to half a unit per required seat, the relaxation vanishes at $n = 2\nu + 1$ for every integer $\nu \geq 2$. The proof combines a single-crossing

argument, exact finite certificates, and an analytic tail bound. It is a computer-assisted result; one seat is an exception.

The probability calculations organize these results but do not supply their implementation. The familiar binomial tail counts sufficient demand under independent willingness; its Gaussian-factor extension is a finite mixture whose limit is the Vasicek expression (Vasicek, 2002). Neither is an arbitrage price. Neither makes a weakly dominant acceptance strategy the outcome of every equilibrium. The distinction is material: at five potential investors and two seats, the interim commitment requirement is zero, but equal terms still support fewer acceptable memberships than efficient selection.

2 Demand that makes formation possible

2.1 Equal terms, subscriptions, and funding

There are n investors and exactly ν identical indivisible seats, with $2 \le \nu \le n$. Forming the program produces a public, distributable resource $V > 0$. A selected investor i incurs private cost c_i and receives payment s_i; his utility is $s_i - c_i$. An unselected investor receives zero. Costs are independent with distributions F_i, unless dependence is specified. These are transferable, quasilinear values in one numeraire. A published term sheet does not reveal an investor's certainty equivalent or make a stochastic investment payoff publicly transferable.

Equal treatment requires a common selected payment s, and the successful-state resource constraint is $\nu s \le V$. Selected participation must be nonnegative for the realized type. No outside subsidy is available for these selected payments. Subscription itself is costless in this subsection. The mechanism posts $s = V/\nu$, accepts binding conditional subscriptions, forms if at least ν investors accept, and selects exactly ν by a preannounced uniform lottery if oversubscribed. Failed and unselected subscriptions receive zero. The choice of lottery is independent of private costs and reports.

Proposition 1 (Equal-treatment demand bound)**.** *Every arrangement satisfying the stated equal-treatment, resource, and selected-participation restrictions forms only when at least ν costs are at most V/ν. Its closing probability is therefore at most*

$$B_{n,\nu}(\pi) = \sum_{k=\nu}^{n} \binom{n}{k} \pi^k (1-\pi)^{n-k}, \qquad \pi = F(V/\nu), \tag{3}$$

under identically distributed independent costs. The posted contract attains this bound when investors accept exactly when $c_i \le V/\nu$, a profile of weakly dominant strategies. Nevertheless, its lowest closing probability across Bayesian Nash equilibria is zero.

The upper bound is ex post feasibility: every selected cost must lie below the common share, itself bounded by V/ν. Acceptance by a willing investor yields either a nonnegative selected payoff or zero; refusal gives zero. For an unwilling investor, refusal weakly dominates acceptance. Ties can accept without changing a continuous-type probability. Universal refusal is also a Bayesian equilibrium because a unilateral subscription cannot fill $\nu \ge 2$ seats. Information disclosure alone does not remove it.

With unequal acceptance probabilities, the same count has a Poisson-binomial distribution. Unequal tickets q_i instead require a weighted event such as $\sum_i q_i \mathbf{1}\{c_i \le s_i\} \ge K$, together with eligibility and

allocation rules. A headcount distribution is not a substitute. Even a full signature set does not imply physical funding. Collectible cash, valid authority, issuer acceptance, and any conditions precedent must still be satisfied at settlement. If the funding event is independent and has probability p_f, actual closing multiplies the subscription probability by p_f; otherwise the joint event must be evaluated.

2.2 Dependence and a finite-pool formula

Let $Z, \epsilon_1, \ldots, \epsilon_n$ be independent standard normal variables. Investor i is willing when $\sqrt{\rho}Z + \sqrt{1-\rho}\epsilon_i \le a$, where $a = \Phi^{-1}(\pi)$ and $0 < \rho < 1$. Conditional willingness is

$$q(z) = \Phi\left(\frac{a - \sqrt{\rho}z}{\sqrt{1-\rho}}\right). \tag{4}$$

Conditional independence gives the exact finite-pool sufficient-demand probability

$$P_{n,\nu} = \int_{-\infty}^{\infty} \phi(z) B_{n,\nu}(q(z))\, dz. \tag{5}$$

If $\nu_n/n \to \alpha \in (0,1)$, the conditional law of large numbers gives

$$P_{n,\nu_n} \longrightarrow \Phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{1-\rho}\Phi^{-1}(\alpha)}{\sqrt{\rho}}\right). \tag{6}$$

This is the Vasicek distribution applied to willingness. Equation 5 retains n and ν separately. The count formula also allows one seat, although the refusal-equilibrium claim requires at least two. At independent willingness $\pi = .4$ and seat ratio $1/2$, the probabilities for pools of 2, 4, and 20 are .64, .5248, and approximately .2447. A ratio-only formula cannot be exact for these pools.

The latent parameter ρ is not the correlation of the binary acceptance indicators. The latter equals $[\Phi_2(a, a; \rho) - \pi^2]/[\pi(1-\pi)]$. Outside a specified dependence model, even marginal acceptance and binary pairwise correlation fail to identify formation. For three investors, assign equal probability to $000, 011, 101, 110$; alternatively assign it to $111, 100, 010, 001$. Both laws have willingness $1/2$ and pairwise correlation zero, but their probabilities of filling two seats are $3/4$ and $1/4$.

More independent search opportunities weakly raise sufficient demand when seats and terms stay fixed. Proportional expansion changes the requirement as well as the pool, so this argument does not apply. In the limit, the correlation derivative has the sign of

$$-\Phi^{-1}(\pi) + \frac{\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}. \tag{7}$$

For example, with $\pi = .29 < \alpha = .30$, raising ρ from .1 to .3 lowers the limiting probability from .4299 to .4171. Being below the required willingness share does not by itself make correlation beneficial. A discrete, independently funded seat backstop has increment $B_{n,\nu-b-1}(\pi) - B_{n,\nu-b}(\pi) = \mathbb{P}\{\mathrm{Bin}(n,\pi) = \nu - b - 1\}$. This compares consecutive integer backstops, not a derivative with respect to cash.

3 The reserve for known willingness

Replace the seat threshold by a nonempty upward-closed family $\mathcal{F} \subseteq 2^N$, with $\varnothing \notin \mathcal{F}$. A membership S forms precisely when $S \in \mathcal{F}$. Every investor pays public expense $h_i \geq 0$ if he subscribes. Every signer has a strictly positive net payoff in every successful membership containing him. If $S \notin \mathcal{F}$, each signer receives the fixed gross bonus $\beta_i \geq 0$ and has net payoff $\beta_i - h_i$. A nonsigner's incremental payoff is zero. Payments are credible, and formation is physical once the membership condition holds. Success-state gains may vary privately, but their strict positivity is common knowledge.

Define the strictly assured set $A(\beta) = \{i : \beta_i > h_i\}$ and maximum failure liability

$$R(\beta) = \max_{S \notin \mathcal{F}} \sum_{i \in S} \beta_i. \tag{8}$$

The reserve covers all possible failed subscriptions, including off-equilibrium ones. A nominal promise unsupported by collectible resources is outside the contract class.

Theorem 2 (Assurance for an upward-closed formation family)**.** *Every equilibrium forms if and only if $A(\beta) \in \mathcal{F}$ or $A(\beta) \cup \{j\} \in \mathcal{F}$ for some $j \notin A(\beta)$. This includes mixed and Bayesian equilibria under the stated pointwise success-gain assumption. The infimum of $R(\beta)$ over implementing contracts is Equation 1. It is attained at zero if some singleton forms; otherwise it is not attained, including when the expense infimum is zero.*

The construction is finite. Every strictly assured investor signs regardless of the others. If their set forms, the result follows. Otherwise one additional investor completes it and strictly prefers signing. All remaining investors then prefer to join as well. If no additional investor can complete it, a profile in which precisely the strictly assured set subscribes is a failed equilibrium. Strictly increasing preferences preserve this argument because each relevant statewise monetary increment has the same strict sign. Positive expected investment value alone would not establish this pointwise comparison under arbitrary risk preferences.

To approach the reserve infimum, choose a minimizing inclusion-minimal forming coalition F, leave a highest-expense member unassured, and give each other member $h_i + \varepsilon$. Their simultaneous liability is due if the unassured member refuses. Conversely, every implementing contract contains a forming coalition all but at most one of whose members are strictly assured. Removing the remaining member exposes a losing coalition whose bonuses exceed its expenses. Appendix A proves both directions and the attainment boundary.

For a ν-of-n rule the answer is the sum of the $\nu - 1$ smallest expenses. For an indispensable cohort it is $\sum_i h_i - \max_i h_i$, the closed-cohort expression in Komkov (2026). The new family formulation accommodates capital thresholds and required combinations of roles. It does not make arbitrary capital thresholds a matroid or give a greedy algorithm for every formation structure.

The reserve has zero equilibrium payout in this known-willingness model. Its financing or carrying cost need not be zero. If an economically available reserve costs rate r for duration τ, $r\tau R$ is a specified carrying-cost approximation, not a theorem about its market price. Investment capital is additional. External cash shortfalls require a different backstop, sized from collectible resources in each funding state. Expiry, transfer limits, or an inability to segregate the reserve can invalidate implementation even when the displayed number is small.

4 Private willingness: assurance must also screen

4.1 An exact closed-cohort characterization

Consider $m \geq 2$ indispensable investors. Types are independent. Investor i is willing with probability $p_i \in (0,1)$; his net successful gain is $g_i > 0$ when willing and $-\ell_i < 0$ otherwise. Both amounts already include the subscription expense h_i. Failure has net payoff $b_i = \beta_i - h_i$, with $\beta_i \geq 0$. Investors observe their own types and simultaneously sign or refuse. All signatures form the program; no report of a hidden type enters a payment. Expected utility is quasilinear over this strategic uncertainty.

Write $A_i = \prod_{j \neq i} p_j$. Exact screening means the strategy profile "sign if and only if willing." It forms with probability $\prod_i p_i$, precisely when an acceptable membership exists for this cohort. The desired property is that *every* Bayesian Nash equilibrium has that strategy profile, up to probability-zero randomizations.

Theorem 3 (Screening and assurance)**.** *The fixed-bonus contract exactly screens willingness in every Bayesian equilibrium if and only if*

$$\#\{i : b_i \leq 0\} \leq 1, \qquad -\frac{A_i g_i}{1 - A_i} < b_i < \frac{A_i \ell_i}{1 - A_i} \quad \text{for every } i. \tag{9}$$

Both inequalities are strict. The claim includes mixed strategies.

The lower and upper bounds come from willing and unwilling incentives at separating play. Sufficiency is stronger than those two incentive constraints alone. All willing types with $b_i > 0$ sign whatever the other investors do. There is at most one remaining willing type; he faces success probability at least A_i and therefore signs by the lower bound. With every willing type signing, each unwilling type faces success probability at least A_i. The upper bound then makes refusing strictly optimal. These three steps apply in every Bayesian equilibrium.

If two net failure payments are nonpositive, those two investors can refuse at every type while every positively assured investor signs at every type. No unilateral deviation forms the indispensable cohort. At an upper-bound tie, an unwilling type can mix into subscription. At a lower-bound tie, after strict upper bounds have been imposed, a willing type can mix into refusal. Thus pure-equilibrium checks alone are inadequate for the exact result.

The known-willingness reserve generally cannot be carried into this model. Define

$$a_j = \max\left\{0, h_j - \frac{A_j g_j}{1 - A_j}\right\}. \tag{10}$$

For each j, let $v_i^{(j)} = h_i$ if $i \neq j$ and $v_j^{(j)} = a_j$.

Corollary 4 (Private-type reserve infimum)**.** *Under the assumptions of Theorem 3, the infimum of maximum failure payments over contracts that exactly screen in every Bayesian equilibrium is*

$$R_{\text{priv}}^{\inf} = \min_j \left\{ \sum_i v_i^{(j)} - \min_i v_i^{(j)} \right\}. \tag{11}$$

Arbitrarily close feasible contracts exist; the infimum is not attained for $m \geq 2$.

All but at most one investor need bonuses strictly above expense. The possible exception still needs enough expected compensation when some other investor is fundamentally unwilling. Given separation, expected gross failure payments equal

$$\mathbb{E}B = \sum_i p_i(1 - A_i)\beta_i. \tag{12}$$

They are now paid on the equilibrium path. For two investors with expenses $(1, 100)$, willing gains $(1, 1)$ and willingness probabilities $(.01, .5)$, the reserve infimum is $9899/99 \simeq 99.9899$ rather than the known-willingness value 1. The example uses positive losses for unwilling types; these losses bound how close to the infimum a particular strict contract can be chosen. With equal expenses h, the infimum remains $(m-1)h$, although the admissible strict margins and expected payments depend on the private-type distribution.

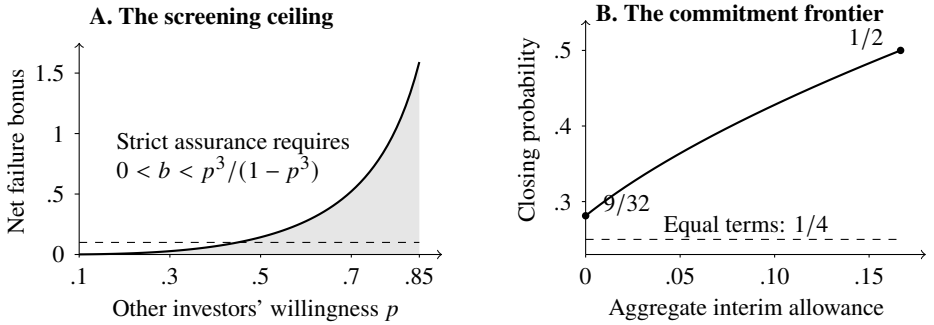


Figure 1: Two different limits on a formation contract

Panel A uses four indispensable investors, independent willingness p, and unwilling loss one. The shaded region is the admissible positive net-bonus interval; its boundaries are excluded. The dashed bonus is .1. Panel B uses two uniform-cost investors, two seats, and $V = 1$, restricting formation to nonnegative real surplus. It plots the highest closing frequency under the interim participation relaxation and an expected budget. That allowance is not an assurance reserve.

4.2 The boundary at a continuous acceptance margin

The finite type gap matters. Suppose each investor has a continuously distributed net successful gain x_i, independently drawn, with positive probabilities on both sides of zero and negative gains arbitrarily close to zero. Failed subscriptions still have the fixed net payoff b_i. In an oversubscribed pool, use a uniform lottery and reimburse unselected subscribers fully so their net payoff is zero. Desired screening is acceptance exactly when $x_i \geq 0$.

Theorem 5 (No fixed-bonus cure at a continuous margin)**.** *For $n \geq \nu \geq 2$ under these assumptions, no fixed nonnegative gross failure-bonus schedule both exactly screens willingness and makes sufficient willing demand lead to formation in every Bayesian equilibrium.*

At separating play, investor i has a positive probability f_i of failure after signing and a positive probability a_i^{sel} of selection. His expected incremental payoff is $a_i^{\text{sel}}x_i + f_i b_i$. If $b_i > 0$, some negative types close enough to zero strictly subscribe. Thus exact screening requires $b_i \leq 0$ for every investor. But then universal refusal is a Bayesian equilibrium. The proof does not rely on Gaussian types or a large pool. A finite gap between willing and unwilling types permits the interval in Theorem 3; a continuous margin closes its positive-bonus side.

Charging unselected subscribers changes the result's premises. Without reimbursement, if an unselected signer loses expense h_i, his expected payoff is

$$a_i^{\text{sel}} x_i + f_i b_i - u_i h_i, \tag{13}$$

where u_i is the unselected probability. For a symmetric three-investor, two-seat pool with willingness $1/2$, $f = 1/4$, $u = 1/12$, and $a^{\text{sel}} = 2/3$. A zero-cutoff incentive constraint then allows $b = h/3 > 0$. With $h = .1$, the gross failure bonus is $2/15$. This supports the specified separating strategy; it does not establish unique implementation. It also exposes an unselected subscriber to a realized loss. Reimbursing that loss restores the zero-net-bonus cutoff and requires cash for reimbursements. The maximum gross obligation in the reimbursed homogeneous model is $\max\{(\nu - 1)\beta, (n - \nu)h\}$, in addition to selected-seat payments. Reimbursement financed from V instead reduces the share available to selected investors. These are different resource arrangements.

5 What an information-rent commitment buys

5.1 Enrollment before the type is learned

Return to independent continuous costs on $[0, \bar{c}_i]$, with positive densities f_i. Investors can enroll before learning their costs; after enrollment, accepted obligations remain enforceable. The allocation selects the ν cheapest costs if their sum S_ν is at most V, and otherwise selects nobody. Investors report costs after observing them. Transfers may depend on these reports through an incentive-compatible mechanism. This is a different contract class from a posted share with failure bonuses.

Let $q_i(c)$ be investor i's interim selection probability. Payment equivalence gives the interim utility under truthful reports as

$$U_i(c) = u_i + \int_c^{\bar{c}_i} q_i(t)\, dt. \tag{14}$$

The constant u_i is utility at the highest cost. Ordinary interim participation requires $u_i \geq 0$. The relaxation permits $u_i \geq -\kappa_i$, $\kappa_i \geq 0$, and requires ex-ante voluntary enrollment, $\mathbb{E}U_i \geq 0$. The budget condition is expected nondeficit: expected payments to investors do not exceed V times the formation probability. State-by-state cash constraints and individual payment limits are not imposed in this relaxation.

Proposition 6 (Minimum aggregate interim relaxation)**.** *For efficient selection under these assumptions, define*

$$I = \mathbb{E}\left[\sum_{i \text{ selected}} \frac{F_i(c_i)}{f_i(c_i)}\right], \qquad W = \mathbb{E}[V - S_\nu]_+. \tag{15}$$

The minimum $\sum_i \kappa_i$ consistent with incentive compatibility, ex-ante voluntary enrollment, and expected nondeficit is

$$\kappa^{\text{inf}} = [I - W]_+. \tag{16}$$

It is attained in the relaxed class by truthful threshold payments with suitable report-independent enrollment charges. Truthful reporting is a weakly dominant strategy after enrollment; the proposition does not guarantee formation in every equilibrium of a larger enrollment game.

Integration of Equation 14 identifies total canonical rents as I. The canonical expected deficit is $I - W$. Constant charges can remove that deficit, and no greater total charge is needed. Ex-ante voluntary enrollment remains possible because $I - W \leq I$. This is the one-dimensional payment-equivalence calculation underlying Krishna and Perry (1998); the note does not claim a new general budget-balance theorem.

The relaxation concerns permission to impose negative interim utility after information arrives. An enrollment charge implements it within this model, but the scalar in Equation 16 is not a formation reserve. For two uniform-cost investors and $V = 1$, efficient threshold payments total $2 - c_1 - c_2$ whenever $c_1 + c_2 \leq 1$. Charging a total $1/6$ balances the budget in expectation. At costs $(0, 0)$, however, payments are 2 against resources $1 + 1/6$, leaving a cash deficit of $5/6$. A funding facility, different timing, or a richer budget-balanced mechanism is still required. The stronger constructions in Krishna and Perry (1998) allow broader transfers; enforceable bounds and liquidity must be checked separately in any financial implementation.

5.2 An exact uniform-cost threshold

With identically distributed uniform costs on $[0, 1]$, $F(c)/f(c) = c$. Thus

$$D_{n,\nu}(V) = I - W = \mathbb{E}[(2S_\nu - V)\mathbf{1}\{S_\nu \leq V\}], \qquad \kappa^{\inf} = [D_{n,\nu}(V)]_+. \tag{17}$$

Table 1 uses exact integration rather than simulation. At four investors the value is $1/40 = .025$. At five, efficient formation has no positive interim-relaxation requirement; equal treatment still loses memberships because some selected costs exceed the equal share even though their sum is below V.

Table 1: Two seats, value one, independent uniform costs

Pool n	Equal terms	Efficient selection	Raw deficit D	$\kappa^{\inf}$
2	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$
3	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{1}{8}$
4	$\frac{11}{16}$	$\frac{7}{8}$	$\frac{1}{40}$	$\frac{1}{40}$
5	$\frac{13}{16}$	$\frac{15}{16}$	$-\frac{1}{12}$	0
6	$\frac{57}{64}$	$\frac{31}{32}$	$-\frac{41}{224}$	0

Equal terms count acceptable memberships at share $1/2$; the posted contract attains this probability under specified weakly dominant play. Efficient selection is a different allocation rule. The last column relaxes interim participation under an expected budget, not the cash reserve needed for either arrangement.

Theorem 7 (Uniform-cost pool threshold; computer-assisted)**.** *For every integer $\nu \geq 2$, at $V = \nu/2$ the first $n \geq \nu$ with $D_{n,\nu}(V) \leq 0$ is $n = 2\nu + 1$. For $\nu = 1$, it is $n = 1$.*

The proof is not extrapolation from a table. Conditional on the νth order statistic, the other selected costs are scaled uniform order statistics. The resulting truncated deficit has one sign change. Increasing pool size reweights toward its negative portion, so the sign can cross at most once. The deficit is negative at $2\nu + 1$. Positivity at 2ν follows from exact rational certificates for $2 \leq \nu \leq 127$ and a bounded-differences inequality

for all $\nu \geq 128$. Appendix A supplies the analytic steps; Appendix B supplies a short rational-arithmetic certificate algorithm. No floating-point comparison is used for the finite signs.

5.3 Closing frequency is a separate objective

Maximizing welfare selects every draw with $S_\nu \leq V$. Maximizing the frequency of closing subject to a tighter expected budget can instead select a subset of those draws. To keep the comparison economic, require no formation when real surplus is negative. For uniform costs, let

$$P(s) = \mathbb{P}(S_\nu \leq s), \qquad D(s) = \mathbb{E}[(2S_\nu - V)\mathbf{1}\{S_\nu \leq s\}], \quad s \leq V. \tag{18}$$

Under the interim relaxation with aggregate allowance k, a highest-frequency rule selects the cheapest ν investors exactly when $S_\nu \leq s$, choosing the largest $s \leq V$ with $D(s) \leq k$. A Lagrange-multiplier comparison selects the lowest virtual deficits first; the resulting allocation is monotone and has truthful threshold-payment implementation. This claims support under truthful reporting, not elimination of all equilibria.

For two investors and $V = 1$, on $3/4 \leq s \leq 1$,

$$P(s) = \frac{s^2}{2}, \qquad k = D(s) = \frac{2s^3}{3} - \frac{s^2}{2}, \qquad \frac{dP}{dk} = \frac{1}{2s-1}. \tag{19}$$

With $k = 0$, the constrained maximum closing probability is $9/32$, compared with $1/4$ under equal terms and $1/2$ efficiently. At $k = 1/6$, the efficient selection probability is reached. Removing the prohibition on negative-surplus formation changes the objective and can produce more closings than the efficient rule. Closing frequency alone is therefore not an adequate measure of an investment institution's performance.

6 Interpretation, scope, and measurement

The results distinguish three objects with different units and assumptions. Sufficient demand is a probability. A credible assurance reserve is a maximum payment due under an observable contingency. An interim participation relaxation measures utility investors agree to surrender at some types after enrolling. None is interchangeable with cash needed to acquire the investment. Their separation is the condition for combining them in a usable contract.

The substantive private-information finding is that assurance can undermine the very screening on which a demand bound relies. A willing investor needs protection against other investors' refusal. An unwilling investor values that same protection but dislikes success. A finite type gap permits bonuses that satisfy both margins; a continuous margin prevents exact screening and all-equilibrium formation within the fixed-bonus class. Conditional subscriptions, sequential offers, type reports, penalties, refundable deposits, and success transfers are distinct instruments whose richer implementation possibilities remain outside that impossibility.

The connection to the flagship is exact. The closed-cohort assurance theorem in Komkov (2026) assumes strictly positive full-sign utility at every type. Theorem 2 extends its resource calculation to upward-closed membership structures while retaining strict successful willingness. Theorem 3 instead admits genuinely unwilling types and derives the screening ceiling. Theorem 5 explains why the demand count cannot simply be appended to the earlier reserve formula. The note leaves the flagship edition unchanged.

The closest priority boundaries are also specific. Tabarrok (1998) already treats excludability, private values, and subscription expenses. Zubrickas (2014) allocates refund bonuses in proportion to contributions, and his known-aggregate-value assumption serves a different information problem. Halac, Kremer, and Winter (2020) characterize optimal success and failure returns under capital constraints using an iterative recruitment argument; their initial-capital extension already insures some investors against failure. The present family result fixes successful terms, allows arbitrary upward-closed formation, and derives an exact maximum failure liability without favorable tie-breaking. Bernstein and Winter (2012) and Segal (2003) analyze recruitment with heterogeneous externalities. Cason, Tabarrok, and Zubrickas (2026) apply refund bonuses to asset assembly experimentally; their positive nonsigner payoff under success permits holdout incentives excluded by the zero-nonsigner-payoff model here. The proposed additions here are the exact family reserve, the private-type screening band and its reserve implication, and the continuous-margin boundary for fixed bonuses. The research ledger distinguishes these mathematical statements from unresolved claims of literature priority.

A real-data formation forecast is a subsequent test. It needs a dated launch-time eligible pool, comparable offers and amendments, terms, signatures, withdrawals, failed raises, reservation expenses, and actual collected funding. Acceptance inferred only after closing is an outcome, not a launch-time predictor. Failure to observe unsuccessful raises creates selection bias. Estimating a Gaussian factor requires identifying both marginal willingness and dependence from repeated comparable opportunities; a single completed deal does not identify either reliably.

AI can affect search reach, willingness at given terms, dependence across alternatives, processing expenses, and reservation duration. These are conditional mechanisms, not established effects of the existing AI state-maintenance experiment. More reach may improve demand while changed outside options reduce willingness. Less documentation expense may lower assurance needs while common processing introduces a shared operational failure. Neither inference accuracy nor a common software provider identifies the correlation parameter in Equation 5.

The present contribution is a formation boundary with contracts attached. Enough willing investors can make an investment feasible without making participation inevitable. A credible failure promise solves that problem when success is wanted; with private willingness, the same promise can attract investors who prefer failure. The appropriate formation law must therefore specify not only who would accept the investment, but which risks the contract pays for and who has committed the resources to pay.

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